ADDITION OF THE SPANWISE EFFECT OF GRAVITY
TO NUMERICAL CALCULATIONS
FOR A SUPERCAVITATING HYDROFOIL OF FINITE SPAN

by

CHARLES GEDNEY

Report No. 83481-1

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Department of Ocean Engineering

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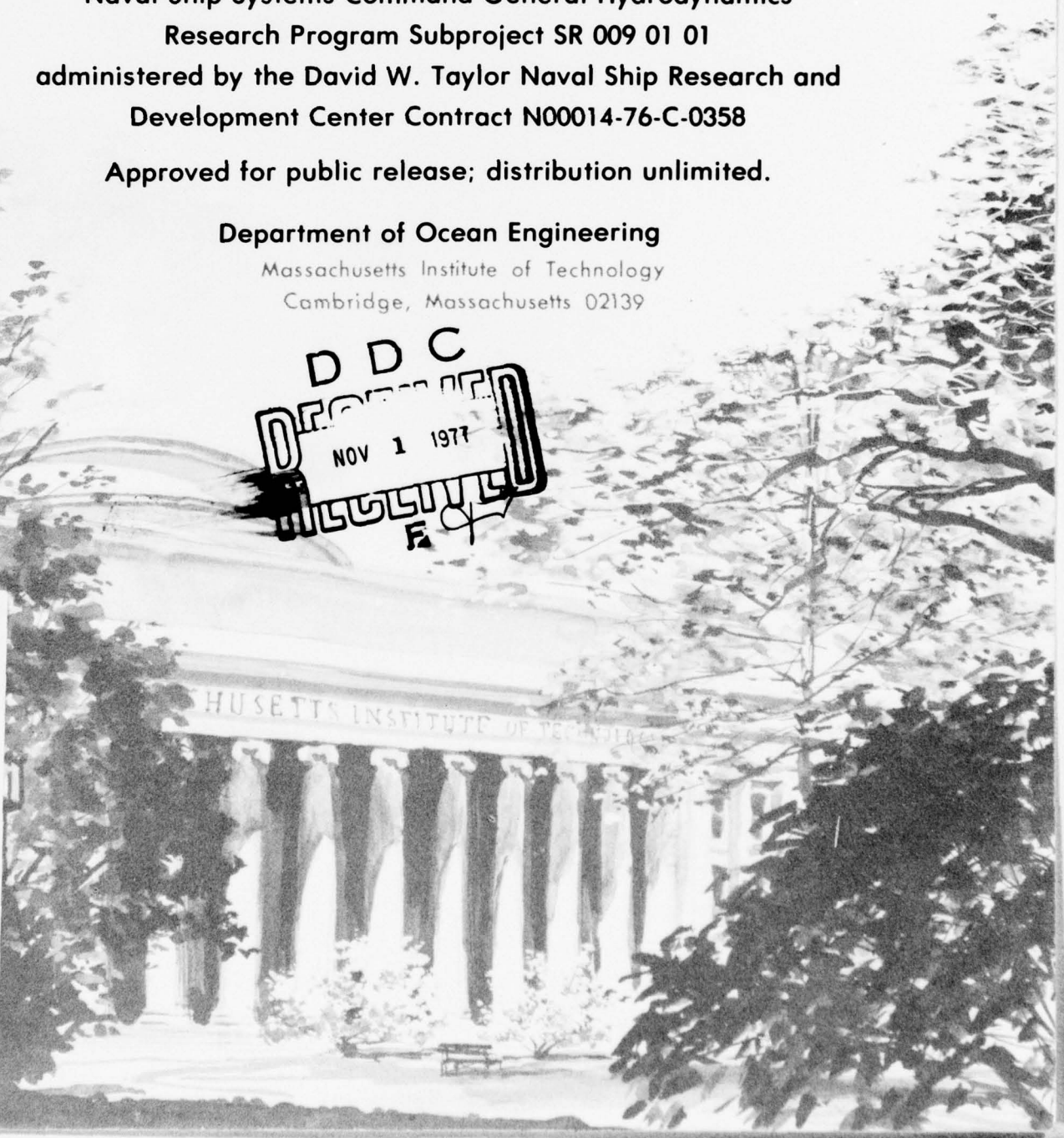CHARLES GEDNEY

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ACKNOWLEDGMENTS

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NOMENCLATURE

c	chord length
C_l	lift coefficient
C_m	moment coefficient
g	accelleration of gravity
h	vertical displacement
ℓ	cavity length
M	number of elements in cavity, chordwise
Ml	number of elements along the foil chord
N	number of elements along the foil span
P_c	cavity pressure
P_∞	free stream pressure (at foil centroid)
q	source strength
R_w	wake surface
S	foil surface
U_∞	free stream velocity
u	perturbation velocity in x direction
v	perturbation velocity in y direction
x_1	leading edge position
x_t	trailing edge position
x,y,z	cartesian coordinates
α	angle of attack
ξ,η,ζ	dummy coordinates for integration

δ_a trailing vortex strength on foil

δ_w trailing vortex strength in wake

γ vortex strength

ρ fluid density

σ cavitation number

Σ cavity surface

I. INTRODUCTION

To insure the accuracy of numerical calculations, they
must be compared to experimental results. In the case of the
three-dimensional supercavitating hydrofoil a numerical model
has been developed and has compared favorably to experimental
results (Jiang and Leehey [1]). However, the experimental
results were obtained using a technique in which the foil was
mounted vertically in a water tunnel with the tip pointed
down (see Fig. 1). This causes a spanwise variation of pressure
and therefore a spanwise variation of cavitation number
($\sigma = (p_\infty - p_c)/\frac{1}{2}\rho U_\infty^2$). The experimental procedure most widely
used was to calculate the cavitation number based on the free
stream pressure at the foil centroid and the measured cavity
pressure (sometimes the vapor pressure is used as the cavity
pressure). It is the intent of this paper to test the effect
of altering the numerical calculation to include these effects.

II. THEORY

A numerical lifting surface model for supercavitating
hydrofoils of finite span has been developed. It is a linear
theory for use on thin wings at small angles of attack. The
model uses discrete vortices and sources located on a lattice
to represent the foil. Each element of the lattice contains
a discrete bound vortex, a trailing vortex and a source. The

bound vortex is located at the quarter chord line of the element and the source is a constant distribution along the three-quarter chord except at the leading edge where the source is located at the quarter chord line. The lattice used in this model is one obtained by dividing the foil and cavity spanwise into strips with cosine spacing and chordwise into strips of constant spacing (see Fig. 2). The solution of the problem is obtained by reducing the coupled integral equations to a set of simultaneous algebraic equations. The cavity lengths were iterated at each chordwise strip to obtain the proper cavitation number over the cavity planform.

The foil and cavity surfaces are collapsed onto the X-Z plane (see Fig. 3). The jumps in tangential and normal pertabation velocity components across the foil and cavity are:

1. $u(x,z,+0) - u(x,z,-0) = -\gamma(x,z)$

2. $v(x,z,+0) - v(x,z,-0) = q(x,z)$

where γ is the vortex distribution and q is the source distribution. The boundary conditions are:

3. $v(x,z,-0) = dy(x,z,-0)/dx$ on the wetted surface of the foil,

4. $u = \sigma/2$ on the cavity,

5. $\int_{x_{\ell}(z)}^{\ell(z)} q(\xi,z)d\xi = 0$ closure,

where the integration is done along each strip of elements from the leading edge to the end of the cavity. The induced velocities are calculated at the midspan, three-quarter chord position of each element in the model. The condition of fixed cavity pressure is also tested in each element, but at the midspan, quarter chord location. The integral equations are:

6. $v(x,z,-0) = -q/2$

$$+ \frac{1}{4\pi} \iint\limits_{S} \frac{\gamma\,(\xi,\eta)\,[x-\xi]+\delta_a(\xi,\eta)\,[y-\eta]}{((x-\xi)^2 + (y-\eta)^2)^{3/2}}\, d\xi d\eta$$

$$+ \frac{1}{4\pi} \iint\limits_{R_w} \frac{\delta_w(\xi,\eta)\,(y-\eta)}{[(x-\xi)^2+(y-\eta)^2]^{3/2}}\, d\xi d\eta$$

7. $$\sigma = -\gamma + \frac{1}{2\pi} \iint\limits_{\Sigma} \frac{q(\xi,\eta)\,[x-\xi]}{[(x-\xi)^2+(y-\eta)^2]^{3/2}}\, d\xi d\eta$$

8. $$\int_{x_\ell(z)}^{\ell(z)} q(\xi,z)\,d\xi = 0$$

with the boundary conditions they become the following set of algebraic equations.

9. $1/4\pi \sum\limits_{i,j} a_{ijk\ell} \gamma_{ij}/\alpha - 1/2\ q_{k\ell}/\alpha = -1$ $i=1,2,\ldots M1$
                                                       $j=1,2,\ldots N$

10. $-\gamma_{k\ell}/\alpha + 1/2\pi \sum\limits_{ij} b_{ijk\ell}q_{ij}/\alpha - \sigma/\alpha = 0$ $i=1,2,\ldots M$
                                                       $j=1,2,\ldots N$
                                                       $\gamma_{k\ell}=0$, for $k>M1$

11. $\sum\limits_{i} q_{ij}/\alpha\ \Delta S_{ij} = 0$ $j=1,2,\ldots N$

where $a_{ijk\ell}$ and $b_{ijk\ell}$ are the v and u velocity components at
the control point (k,ℓ), caused by a unit strength Vortex
and source at element (i,j). M is the number of elements
along the chord in the cavity and M1 and N are the number of
elements on the foil in the chordwise and spanwise directions,
respectively. The accuracy of this model has been tested by
Jiang and Leehey [1] and results compare well with both
analytical and experimental results (see also figures 4-6).

    To include the spanwise effects of gravity for the foil
in the verticle (experimental) position, equations (4) and (10)
above must be changed to:

4'. $\ u = \sigma(z)/2$

10'. $-\gamma_{k\ell}/a + 1/2\pi \sum\limits_{i,j} b_{ijk\ell}q_{ij}/\alpha - \sigma_\ell/\alpha = 0$ $i=1,2,\ldots M$
                                                                 $j=1,2,\ldots N$
                                                                 $\gamma_{k\ell}=0$ for $k>M1$

where σ_ℓ is now a function of span position (ℓ). The change
in free stream pressure ΔP_∞ due to a change in vertical
position h is given by:

12. $\Delta P_\infty = \rho gh$ (see Fig. 1).

Therefore the change in cavitation number, $\Delta\sigma$ due to a change in position, h is given by:

13. $\Delta\sigma = (P_\infty' - P_C)/\frac{1}{2} \rho U_\infty^2 - (P_\infty - P_C)/\frac{1}{2} \rho U_\infty^2$

  $= \Delta P_\infty/\frac{1}{2} \rho U_\infty^2 = 2gh/U_\infty^2$.

Experimental procedures quote cavitation numbers based on free stream pressures measured at the centroid of the foil. Therefore the variation of cavitation number over the span can be determined using equation (13) and the "overall" cavitation number (at the centroid of the elliptical planform).

III. RESULTS

Equation (13) was added to the numerical technique and the input cavitation number was taken as the corresponding to the foil centroid. Then, an array of cavitation numbers, each corresponding to the center of a strip of elements along the chord of the foil were determined (see Fig. 2). The model was also changed so that equation (10') was used in place of equation (10). Several test conditions were calculated and the results are plotted in Figures 4-7. The test conditions used were for two foils of aspect ratio five and elliptical

planform, each at four different values of cavitation number divided by angle of attack (σ/α). One foil had a semi-span of 10 inches, the other was 15 inches. (Experimental results are available for foils of these sizes from Maixner [3], these are shown for comparison. Analytical results of Leehey [4] are also shown.) The values of σ/α used were 1.0, 1.5, 2.0 and 2.5. The results show that the spanwise effect of gravity is negligible in this model. Both lift coefficient over angle of attack (C_l/α) (Fig. 4) and moment coefficient over angle of attack (C_m/α) (Fig. 5) increased in magnitude over the range of σ/α tested, but the increase was only in the third significant figure. The larger foil had larger values of C_l/α and C_m/α, but again the differences were small. The cavity length to chord ratio (ℓ/c) (see Fig. 6) was larger than the original numerical model by about 0.2 for high σ/α and shorter by about 0.2 for low σ/α, but both models agree well with experimental data. Figure 7 shows the major difference between the two models, the cavity shape. The cavity shape is plotted for the small foil at $\sigma/\alpha = 1.5$ as it was calculated by the two numerical methods. The new model obtains a shorter cavity near the tip due to the higher cavitation number there and a longer cavity near the root due to the lower cavitation number. The experimental result for this condition (from Maixner [3]) is also shown. Figure 8 is the photograph used to determine this result.

The experimental result compares favorably with the new model
near the tip but the cavity is shorter near the root due to
the boundary layer on the upper wall of the test section.

IV. CONCLUSIONS

It is clear from the results presented here that the
spanwise effect of gravity can be neglected for the numerical
model. The effect of including the spanwise variation in
cavitation number on lift and moment calculations has been
shown to be only on the order of one percent. The cavity
length calculations changed by about five percent (that is
when the cavity length is measured at the span position of the
centroid). The most striking difference found in the
numerical results when the spanwise variation in cavitation
number was taken into account was in the cavity shape predicted.
This shows that the reason for the insignificant differences
in the models is that the cavities predicted are about the same
length near the centroid, where the cavitation number was
measured in experimental work.
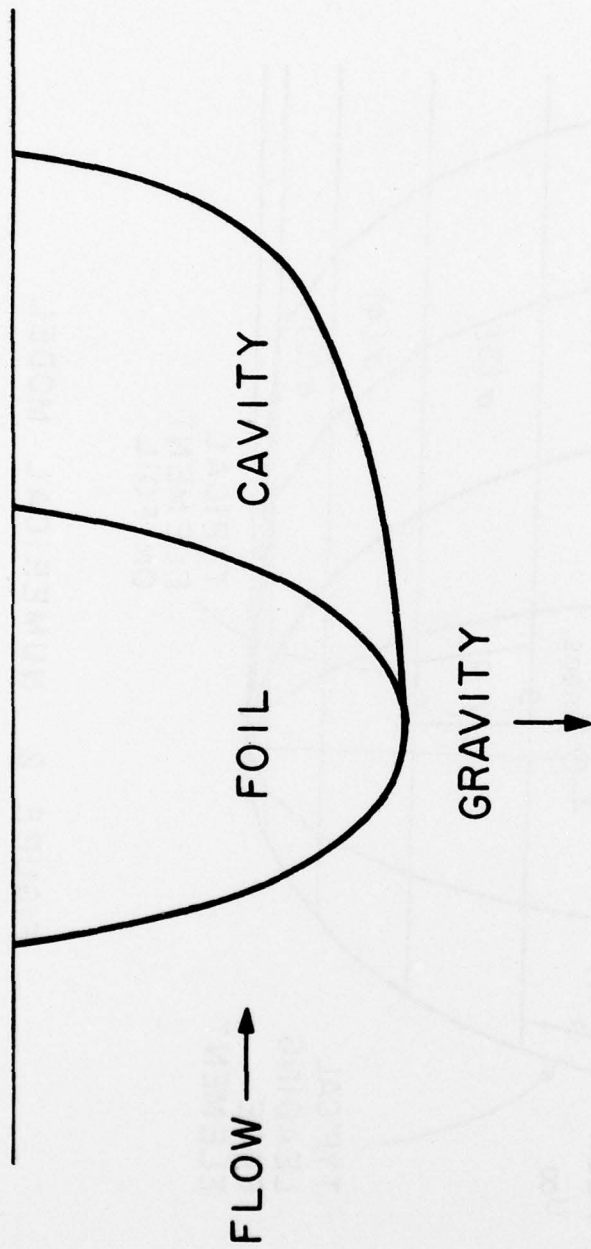
FIGURE I EXPERIMENTAL SETUP
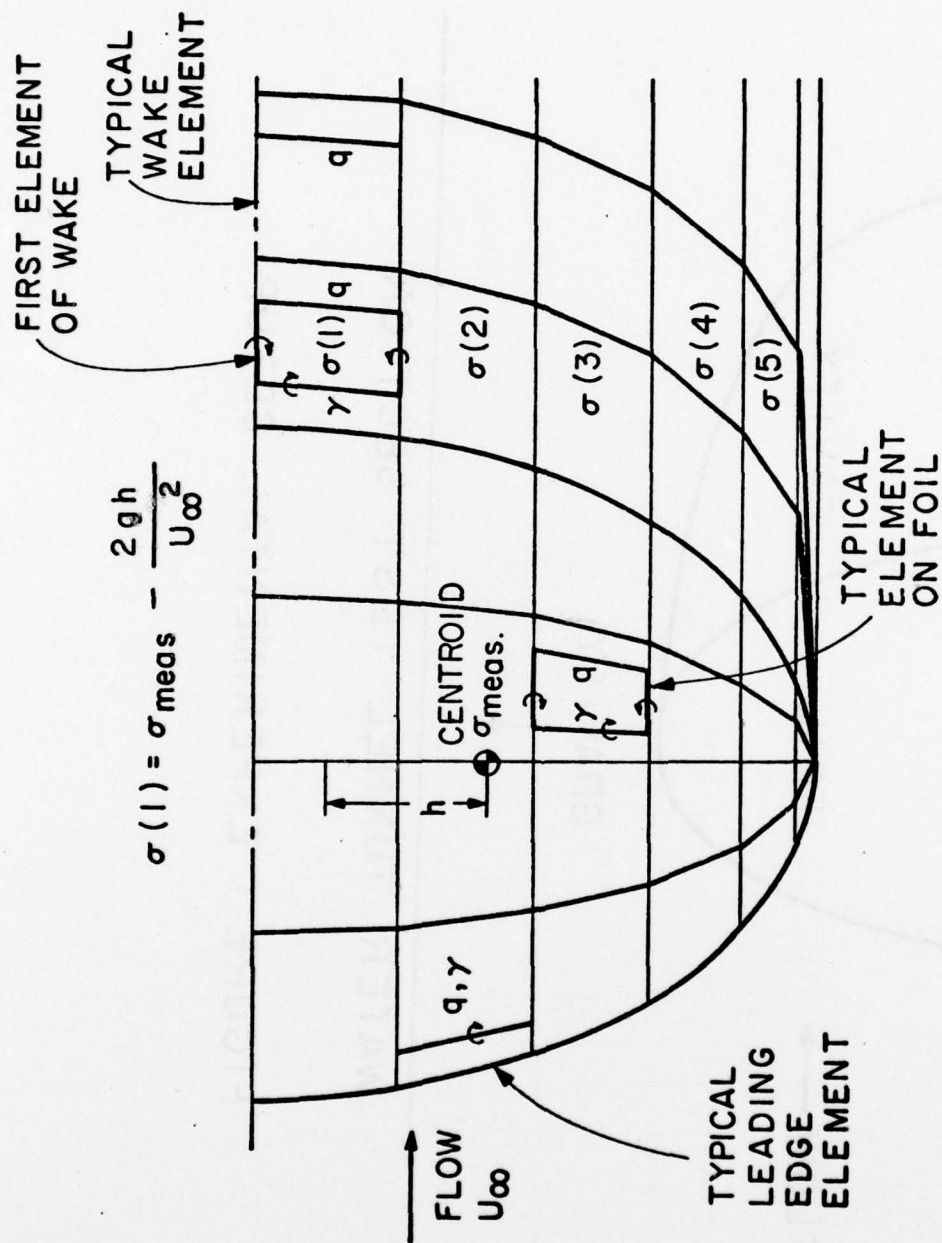
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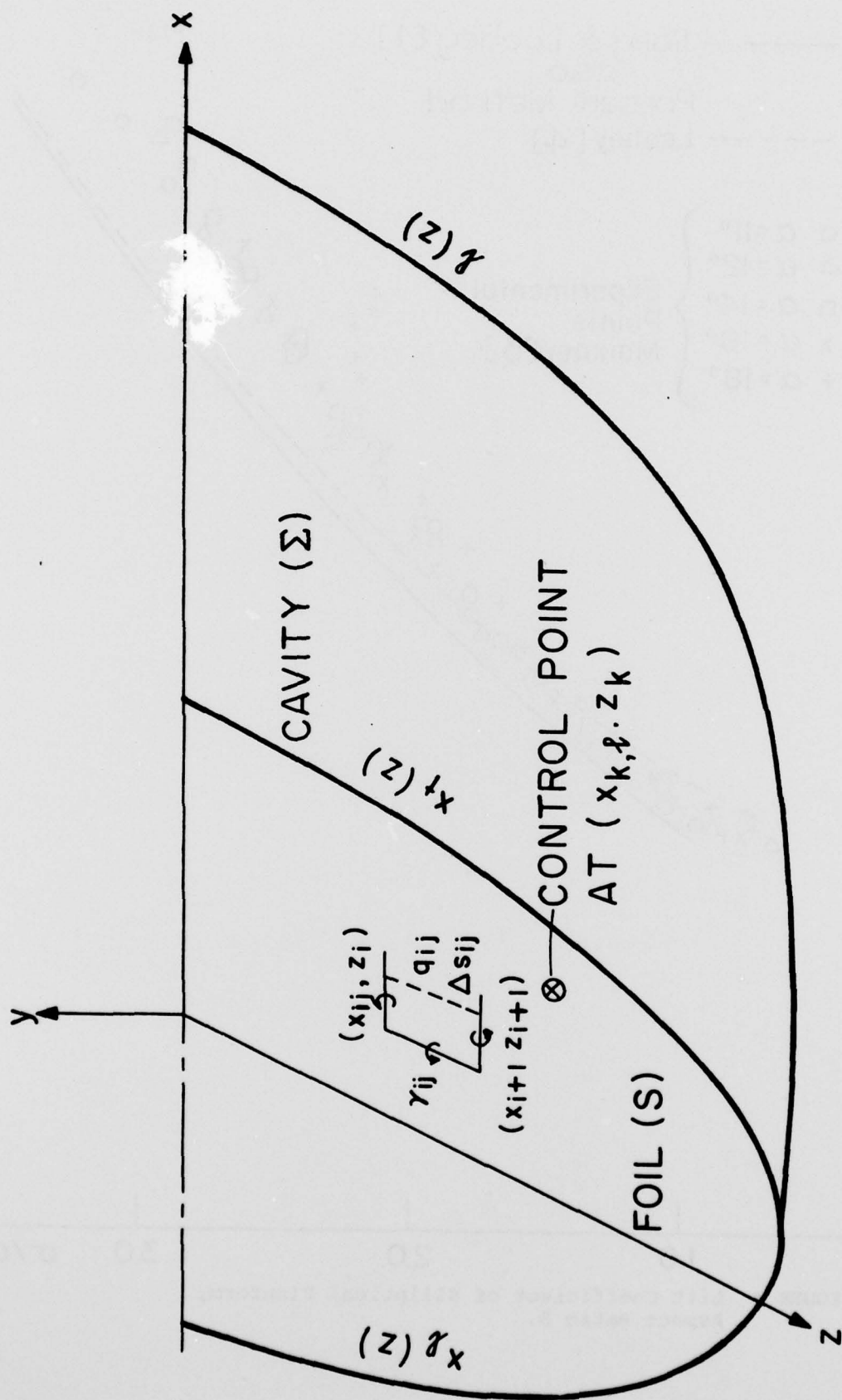
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ELEMENT
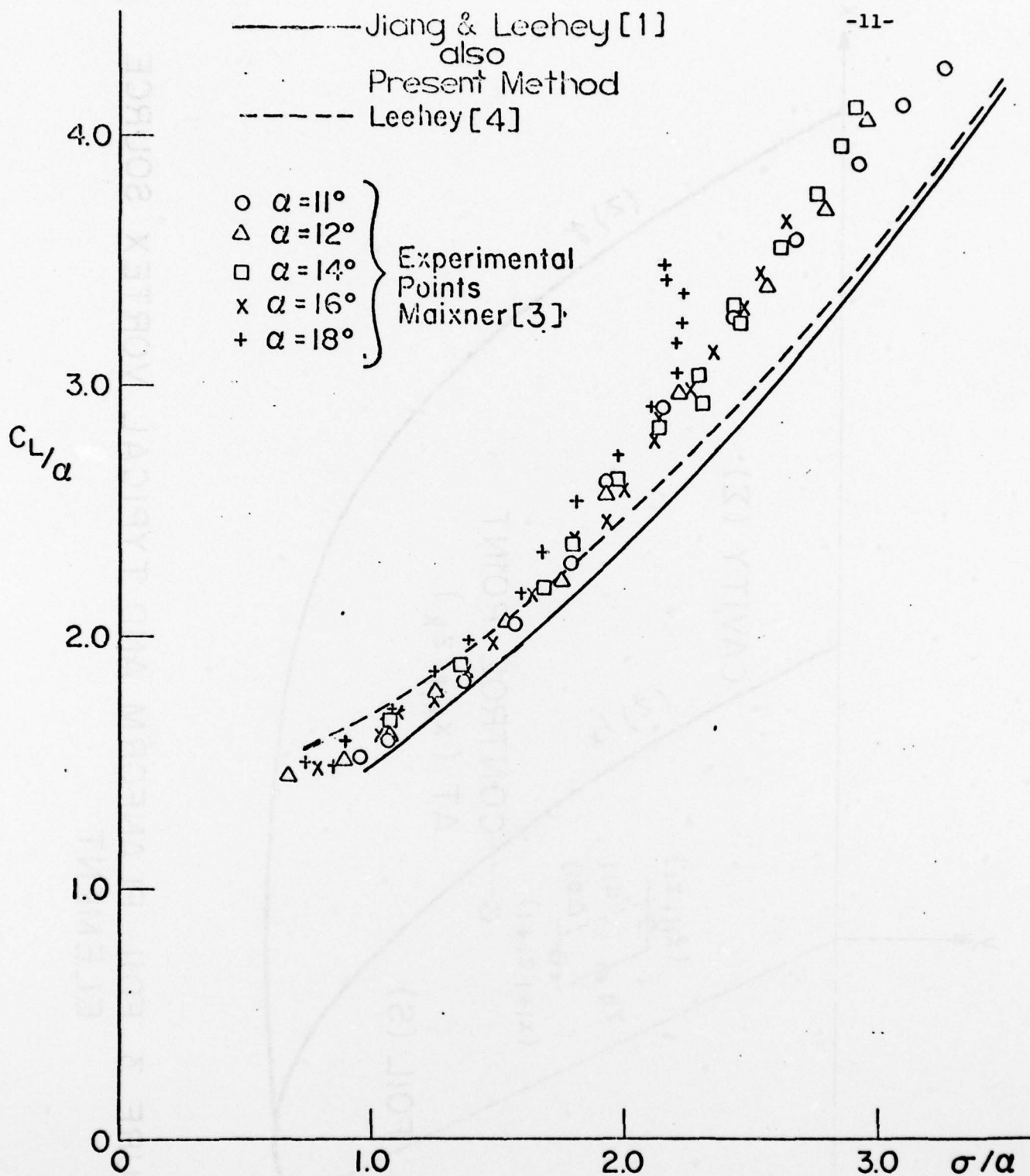
FIGURE 4   Lift Coefficient of Elliptical Planform,
Aspect Ratio 5.
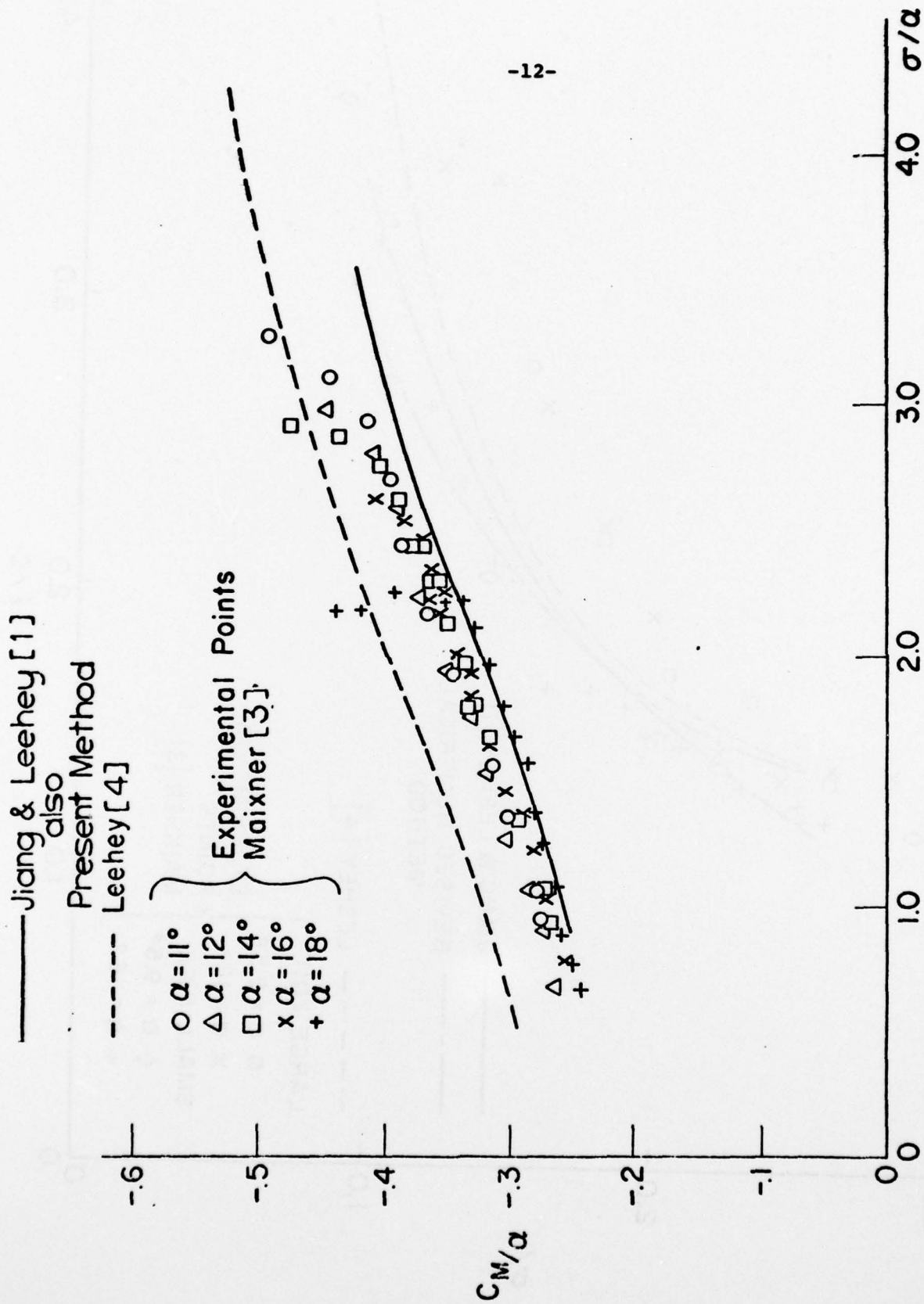
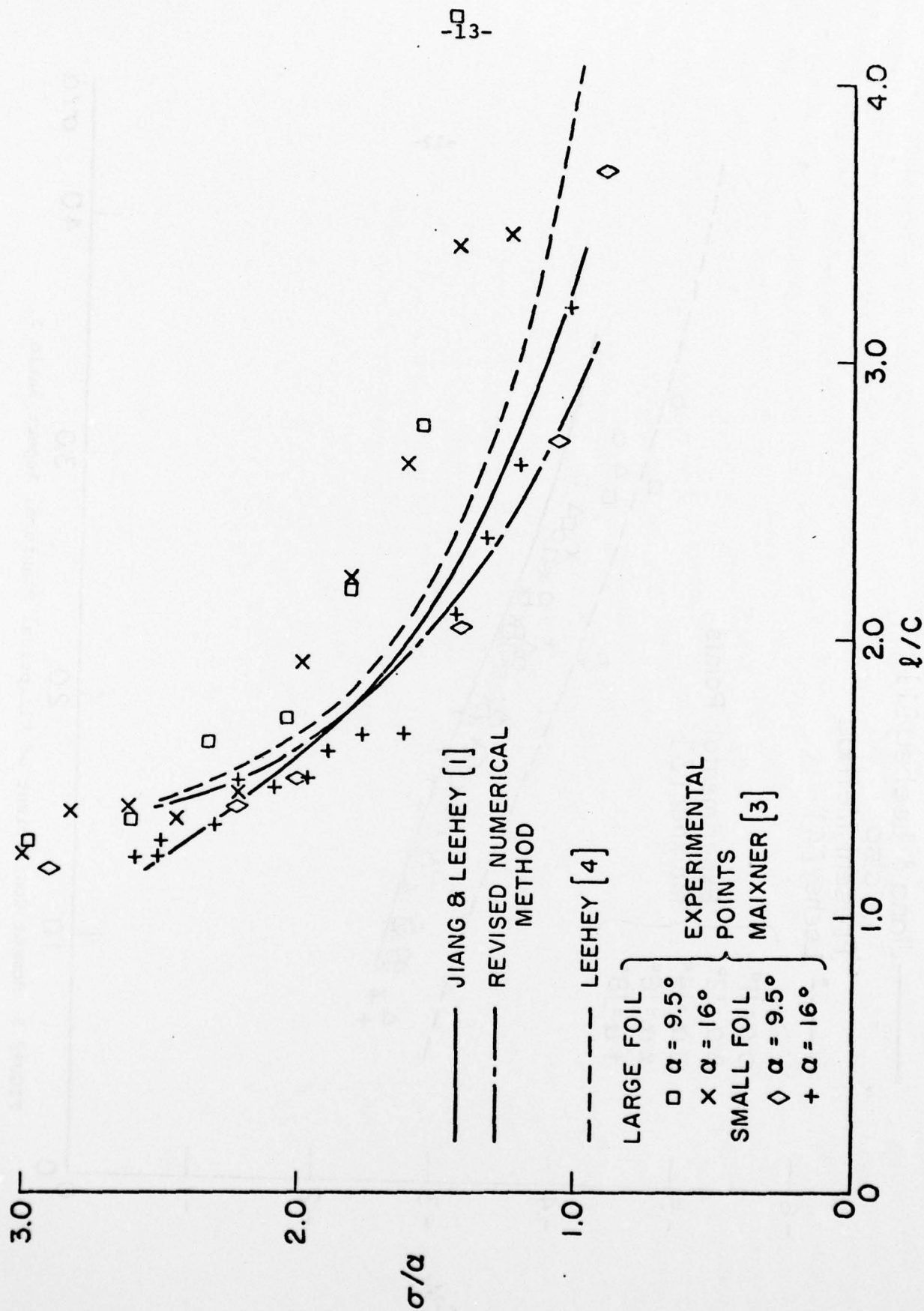FIGURE 5 Moment Coefficient of Elliptical Planform, Aspect Ratio 5.

FIGURE 6 Cavity Length to Chord Length Ratio of Elliptical Planform, Aspect Ratio 5.
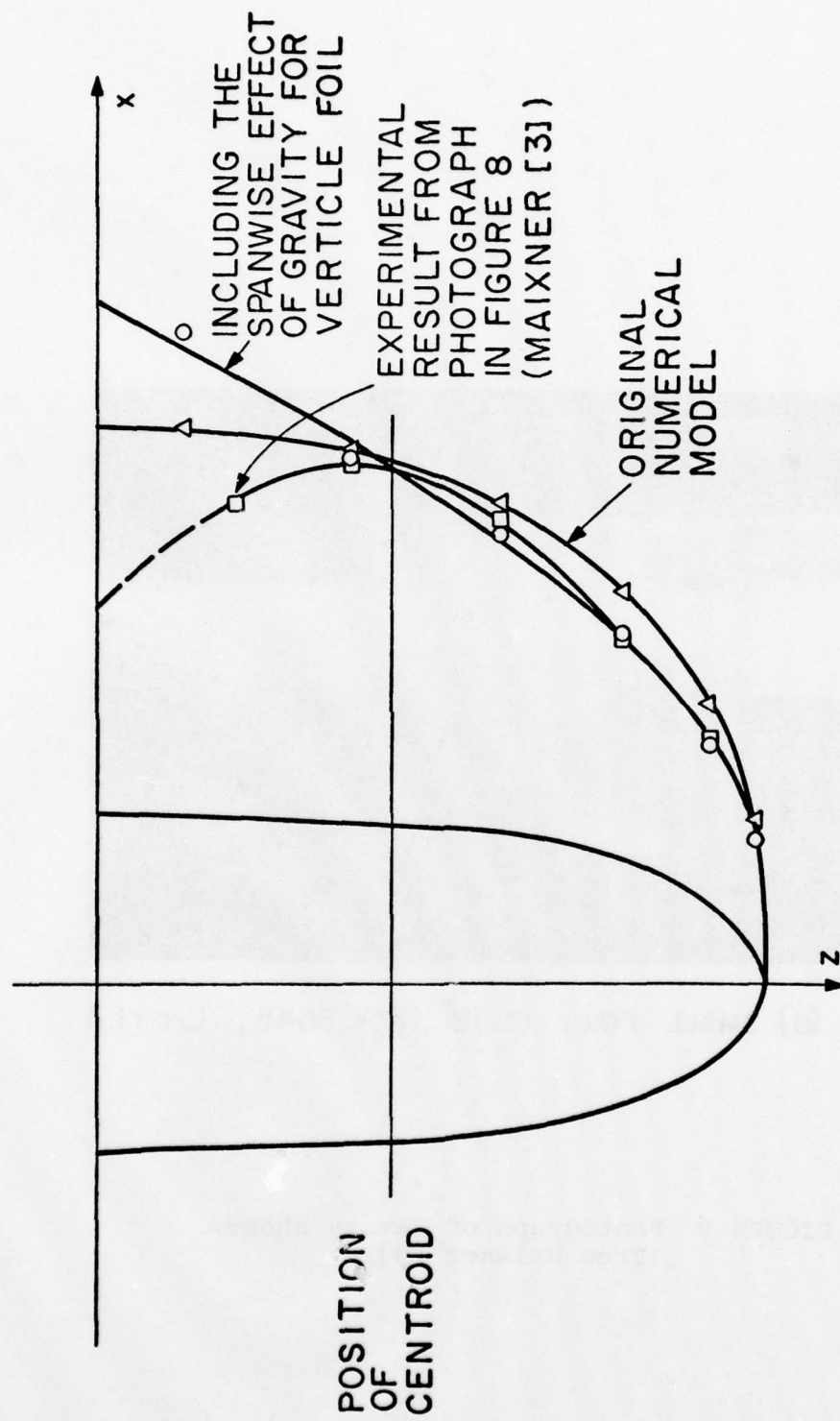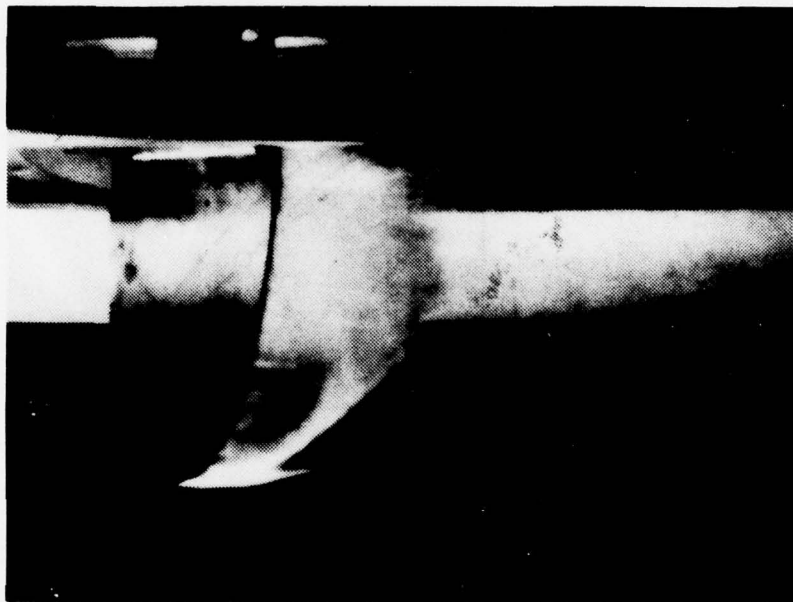
INCLUDING THE
SPANWISE EFFECT
OF GRAVITY FOR
VERTICLE FOIL

EXPERIMENTAL
RESULT FROM
PHOTOGRAPH
IN FIGURE 8
(MAIXNER [3])

ORIGINAL
NUMERICAL
MODEL

POSITION
OF
CENTROID

x

z

FIGURE 7   CAVITY SHAPES, ASPECT RATIO 5, $\sigma/\alpha = 1.5$
SEMI – SPAN = 10"   SCALE: 1/2 SIZE

(a) SMALL FOIL: $\alpha = 12°$, $\sigma_c = .3545$, $L/c = 1.3$

FIGURE 8   Photograph of Cavity Shape.
(From Maixner [3]).

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